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LETTER TO THE EDITOR

On wave propagation in fluids of polyatomic molecules

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Abstract. Making use of the Müller theory of irreversible processes, some solutions of the momentum and internal angular momentum balance equations of a fluid of polyatomic molecules are studied. When one considers only dissipative phenomena described by the rotational viscosity, it is seen that even in the case in which the spin and the vorticity coincide we can have normal mode solutions for the velocity and spin.

In the study of fluids of polyatomic molecules it is necessary to introduce some new hydrodynamical variables in order to take account of the internal state of each molecule. The consequence of the structure of the molecules is that the pressure tensor is not symmetric and therefore the angular momentum is not conserved. So one introduces a new angular momentum, often called the internal angular momentum (de Groot and Mazur 1969), which can be written

$$\boldsymbol{S} = j\boldsymbol{\omega}, \tag{1}$$

where j is the mean moment of inertia and $\boldsymbol{\omega}$ is the spin, responsible for the internal rotations. The balance equations of the momentum and of the internal angular momentum of a fluid with spin (Snider and Lewchuk 1967) or a micropolar fluid (Eringen 1966) are respectively

$$\rho\dot{\boldsymbol{v}} + \nabla \cdot \mathbf{P} = \boldsymbol{F}, \tag{2}$$

$$\rho j\dot{\boldsymbol{\omega}} + \nabla \cdot \mathbf{\Omega} = -2\boldsymbol{P}^{\mathrm{va}} \cdot \boldsymbol{\omega} + \boldsymbol{N}, \tag{3}$$

where ρ is the density, $\boldsymbol{v}$ is the velocity, $\mathbf{P}$ is the pressure tensor, $\mathbf{\Omega}$ is the spin flux, $\boldsymbol{P}^{\mathrm{va}}$ is the axial vector related to the antisymmetric part of the pressure tensor, and $\boldsymbol{F}$ and $\boldsymbol{N}$ are the external force and external couple respectively. An upper dot stands for material derivation.

In order to solve the differential system (2) and (3) it is necessary to introduce constitutive equations for the fluxes that appear in such equations. In the framework of the local equilibrium hypothesis these constitutive equations can be obtained from a Gibbs equation in the same way that one proceeds with classical fluids (Baranowski and Romotowski 1964, Snider and Lewchuk 1967). On the basis of the Müller theory (Müller 1967, Jou *et al* 1979, Lebon *et al* 1980) it is possible to obtain a set of constitutive equations which include temporal derivatives of the fluxes and non-linear terms. In the linear approach these equations reduce to that obtained with the local equilibrium hypothesis (Rubí and Casas-Vázquez 1980).

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The purpose of this Letter is to study some solutions of the balance equations (2) and (3) and their implication in the decay of the spin towards the vorticity. For the sake of simplicity we focus attention on fluids with viscous dissipative phenomena due only to the difference between the spin and the vorticity. Furthermore we consider that the spin flux and the pressure gradient vanish, and likewise convective effects are neglected. In this case equations (2) and (3) can be rewritten

$$\rho\dot{\boldsymbol{v}} = -\eta_r \operatorname{curl}(\operatorname{curl}\boldsymbol{v} - 2\boldsymbol{\omega}) - \eta_r\epsilon\rho\ddot{\boldsymbol{v}}, \tag{4}$$

$$\rho j\dot{\boldsymbol{\omega}} = 2\eta_r(\operatorname{curl}\boldsymbol{v} - 2\boldsymbol{\omega}) - \eta_r\epsilon\rho j\ddot{\boldsymbol{\omega}}, \tag{5}$$

where η_r is the rotational viscosity, and ϵ is defined through the expression (Rubí and Casas-Vázquez 1980)

$$\partial s/\partial \boldsymbol{P}^{va} = -\rho^{-1}T^{-1}\epsilon\boldsymbol{P}^{va}, \tag{6}$$

T being the absolute temperature. The quantity $\eta_r\epsilon$ can be identified with a relaxation time τ^* for the propagation of the signals due to the difference between the spin and the vorticity. A trivial solution of (4) and (5), with $\tau^* = 0$, can be obtained in the case in which curl $\boldsymbol{v}$ is uniform and constant and the spin vanishes initially (de Groot and Mazur 1969). Thus the spin can be written

$$\boldsymbol{\omega} = \tfrac{1}{2}\operatorname{curl}\boldsymbol{v}[1 - \exp(-t/\tau)], \tag{7}$$

τ being a relaxation time equal to $\rho j/4\eta_r$. Equation (7) shows that when $t \to \infty$, $\boldsymbol{\omega} \to \frac{1}{2}\operatorname{curl}\boldsymbol{v}$, and therefore the fluid can be described by means of the classical Navier–Stokes equation. When $\tau^* \neq 0$, the solution for the spin is (Rubí and Casas-Vázquez 1980)

$$\boldsymbol{\omega} = \boldsymbol{A}\exp(r_+t) + \boldsymbol{B}\exp(r_-t) + \tfrac{1}{2}\operatorname{curl}\boldsymbol{v}, \tag{8}$$

$\boldsymbol{A}$ and $\boldsymbol{B}$ being two constants and $r_\pm$ being given by

$$r_\pm = \tfrac{1}{2}\epsilon[-1/\eta_r \pm (1/\eta_r^2 - 16\epsilon/\rho j)^{1/2}]. \tag{9}$$

In the same way as Kranyš (1977) we consider here solutions of equations (4) and (5) of the form

$$\boldsymbol{v}(\boldsymbol{r}, t) = \boldsymbol{v}_{\nu k}\exp[-\mathrm{i}(\nu t - \boldsymbol{k}\cdot\boldsymbol{r})], \tag{10}$$

$$\boldsymbol{\omega}(\boldsymbol{r}, t) = \boldsymbol{\omega}_{\nu k}\exp[-\mathrm{i}(\nu t - \boldsymbol{k}\cdot\boldsymbol{r})], \tag{11}$$

where ν is the frequency (in general, complex) and $\boldsymbol{k}$ the wavevector. Inserting (10) and (11) in (4) and (5) one obtains the relation

$$\boldsymbol{v}_{\nu k} = -\tfrac{1}{2}\mathrm{i}j(\boldsymbol{k}\times\boldsymbol{\omega}_{\nu k}). \tag{12}$$

Equations (4), (5), (10) and (11) with the wavevector such that $\boldsymbol{k} = (k, 0, 0)$ lead to the dispersion equation

$$\eta_r\epsilon\rho j\nu^2 - 4\eta_r + \nu\rho j\mathrm{i} = \eta_r jk^2. \tag{13}$$

In the limit case when the relaxation time τ^* vanishes, one obtains the dispersion equation

$$\nu = -(\eta_r/\rho j)(4 + jk^2)\mathrm{i}, \tag{14}$$

which describes the rotational mode. In any other case (13) leads to

$$\nu = -(1/2\tau^*)\{1 + [1 + 4\eta_r\tau^*(4 + jk^2)/\rho j]^{1/2}\}\mathrm{i}. \tag{15}$$

Of interest is the case in which the spin and the vorticity coincide. From equations (4), (5), (10) and (11) we have

$$\nu = -(1/\tau^*)\mathrm{i}. \tag{16}$$

This equation indicates that even in this case one can have solutions like (10) and (11) which decay to zero depending on the value of τ^*. As pointed out by some authors (Ailawadi and Harris 1972, Pomeau and Weber 1973), since the total angular momentum must be conserved, any internal angular momentum communicated to a molecule creates a vortex which decays until the equilibrium is established. Equation (16) shows that, considering the fact that the differences between the spin and the vorticity propagate with finite speed, the former process must be modified.

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